Grove Road Primary School

Calculations Policy

Policy Review	
Review Schedule	Every 3 years
Reviewing Committee	Teaching and Learning
Date of Last Review	February 2017
Date of Next Review	February 2020

Head Teacher Signature	**Date Signed**
Chair of Governors Signature	**Date Signed**

Grove Road Primary School - Calculation Policy

Updated January 2017, in line with the revised National Curriculum

Introduction

At Grove Road Primary School we believe that children should be introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with a range of efficient methods and strategies that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

The overall aim is that when children leave Grove Road Primary School they:

- have a secure knowledge of number facts (including number bonds and multiplication tables) and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;
- use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

Mental methods of calculation

Oral and mental work in mathematics is essential, particularly so in calculation. Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts. Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied. Ongoing oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a 'feel' for numbers is the product of structured practice and repetition. It requires an understanding of number patterns and relationships developed through directed enquiry, use of models and images and the application of acquired number knowledge and skills. Secure mental calculation requires the ability to:

- recall key number facts instantly – for example, all addition and subtraction facts for each number to at least 10 (Year1/2), sums and differences of multiples of 10 (Year 2/3) and multiplication facts up to 12 × 12 (Year 4). (See National Curriculum 2014 for further details of expectations)

Written methods of calculation

The aim is that by the end of Key Stage 2, the great majority of children should be able to use an efficient written method for each operation with confidence and understanding. This guidance promotes the use of what are commonly known as ‘standard’ written methods – methods that are efficient and work for any calculations, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. We want children to know that they have such a reliable, written method to which they can turn when the need arises.

In setting out these aims, the intention is that we adopt greater consistency in our approach to calculation. The challenge is for our teachers to determine when their children should move on to a refinement in the method and become confident and more efficient at written calculation.

Children should be equipped to decide when it is best to use a mental, written or calculator method based on the knowledge that they are in control of this choice as they are able to carry out all three methods with confidence.

Talking:

- There is evidence that peer interactions are a main facilitator factor for socio-cognitive development and their performance in mathematical tasks.
- A classroom culture of questioning in which pupils learn from shared discussions with teachers and peers.
- The teacher’s role is to help pupils to feel free and confident to comment and to suggest strategies even if there are errors in calculations. This allows for all pupils to reflect and suggest more appropriate methods.
- To encourage teachers to value all contributions and be willing to change their own minds in the light of what the pupil says.
- Tasks are planned so there are opportunities for pupils to communicate their evolving understanding.

We value the communication between teachers and pupils, pupils and their peers. When children feel confident to communicate their ideas and discuss their findings openly this improves their level of understanding. It has been proved that children will remember 70% of what they have been learning if they taken an active part in the lesson compared to a passive learner who will only retain 20% of what has been taught.

Vocabulary:

Vocabulary builds as the children progress through the school and the vocabulary highlighted for each year group are the most frequently used words.

Choosing the appropriate strategy

Recording in mathematics, and in calculation in particular is an important tool both for furthering the understanding of ideas and for communicating those ideas to others. A useful written method is one that helps children carry out a calculation and can be understood by others. Written methods are complementary to mental methods and should not be seen as separate from them. The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. It is important children acquire secure mental methods of calculation and one efficient written method of calculation for addition, subtraction, multiplication and division which they know they can rely on when mental methods are not appropriate. As a long term aim children should be able to choose an efficient method that is appropriate to a given task.

Please see the revised National Curriculum for 2014 for the statutory objectives for the four operations by year group. The suggested phases below are designed to support these objectives.

Written methods for addition

These phases show the building up to using an efficient written method for addition of whole numbers and decimals.

To add successfully, children need to be able to:

- recall all single digit addition pairs to 9 + 9 and number bonds to 10 (making ten e.g. 4+6=10);
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into 70 + 4 or 60 + 14)..

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

Progression in addition and subtraction

Addition and subtraction are connected.

Part	**Part**
Whole	

Addition names the whole in terms of the parts and **subtraction** names a missing part of the whole.

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Phase 1

Develop secure one-one correspondence and understanding of addition.

Combing two sets of numbers (aggregation)

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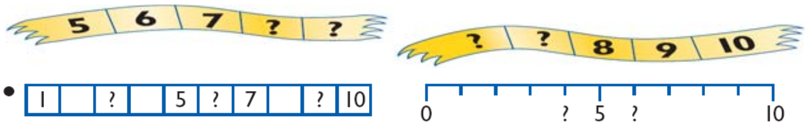
- Count accurately 0-10
- Recognise and write numerals 1-10

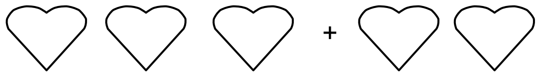


What is the number before 5? And after 5?
Before 10? What is the number between 3 and 5?

- What numbers are between 7 and 10?

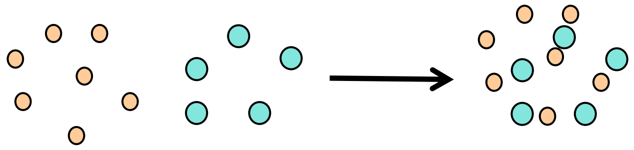
- Count and add together sets of real objects and pictures.

3+2 = 5



- Putting together – two or more amounts or numbers are put together to make a total

7 + 5 = 12



- Count one set, then the other set. Combine the sets and count again. Starting at 1.
- Counting along the bead bar, count out the 2 sets, then draw them together, count again. Starting at 1.

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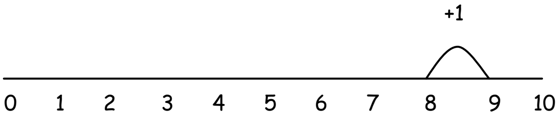
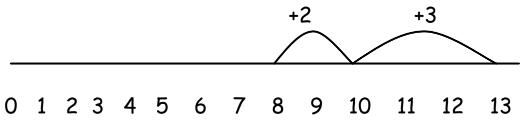
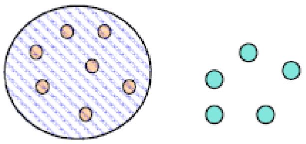
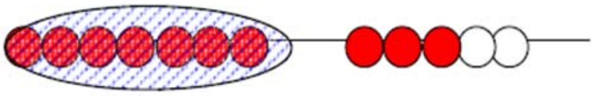
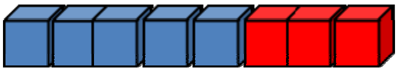
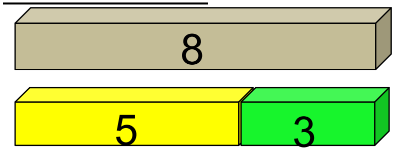
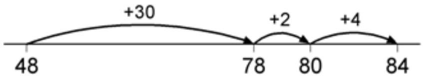
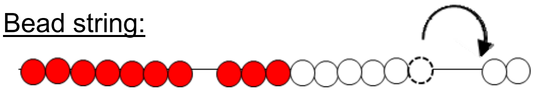
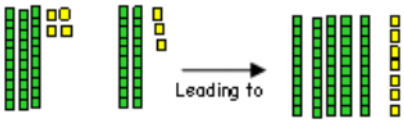
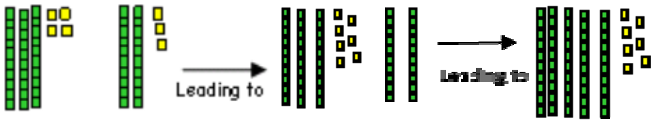
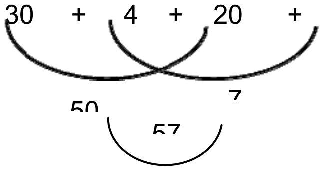
Phase 2: The number line and 100 square	**Phase 2**	**Key vocabulary**
The number line helps children to move from using concrete objects.	Number line 8+1 = 9  - To be able to add through 10.	Add, more, count on, plus, sum, total, altogether, partition, how many. Multiple of 10,number line, 100 square
Children begin to split a number to add to the nearest multiple of 10 and then count on.	8+5 = 13 	
The 100 square supports children's understanding when adding ten to any number the units stay the same and the tens go up. Eventually this will be done mentally.	100 Square To be able to add 10 to any number up to 100 using a 100 square by initially counting on 10. 9+10 = 10+9 = 19 To be able to add multiples of 10 to any number up to 100 using a 100 square. 34+40 = 74	
Combining two sets (augmentation) *This stage is essential in starting children to calculate rather than counting* Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.	Counters:  Start with 7, then count on 8, 9, 10, 11, 12 Bead strings:  Make a set of 7 and a set of 5. Then count on from 7. Multilink Towers:  Cuisenaire Rods:  Number tracks: 	

Phase 3: The compensation model using an empty number line and bead strings - The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and units separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and units to help them make multiples of ten by adding in steps. - The empty number line helps to record the steps on the way to calculating the total.	**Phase 3** <u>Empty number line:</u> The steps often bridge through a multiple of 10. 8 + 7 = 15  To be able to add 2 two digit numbers on an empty number line. 48 + 36 = 84  or:  <u>Bead string:</u>  Children find 7, then add on 10 and then adjust by removing 1.	***As above***

Phase 4: Partitioning • The next stage is to record mental methods using partitioning. Add the tens and then the units to form partial sums and then add these partial sums. • Partitioning both numbers into tens and units mirrors the column method where units are placed under units and tens under tens. This also links to mental methods.	**Phase 4** Record steps in addition using partitioning: 47 + 76 = 47 + 70 + 6 = 117 + 6 = 123 or 47 + 76 40 + 70 = 110 7 + 6 = 13 = 123 Partitioned numbers are then written under one another: $\begin{array}{rcl} 47 & = & 40 + 7 \\ +\underline{76} & & \underline{70 + 6} \\ & & 110 + 13 = 123 \end{array}$ Equipment can also be used to support with children's understanding: <u>Partitioning (Aggregation model)</u> 34 + 23 = 57 <u>Base 10 equipment:</u>  Children create the two sets with Base 10 equipment and then combine; ones with ones, tens with tens. **<u>Partitioning (Augmentation model)</u>** <u>Base 10 equipment:</u> Encourage the children to begin counting from the first set of ones and tens, avoiding counting from 1. Beginning with the ones in preparation for formal columnar method.  <u>Number line:</u>  At this stage, children can begin to use an informal method to support, record and explain their method. (optional) 30 + 4 + 20 + 3 	***As above*** column, addition, tens boundary **Teaching point** Ensure correct use of the = sign.

Phase 5: Expanded method in columns

- Move on to a layout showing the addition of the tens to the tens and the units to the units separately. To find the partial sums either the tens or the units can be added first, and the total of the partial sums can be found by adding them in any order. As children gain confidence, ask them to start by adding the units digits first always.
- The addition of the tens in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'.
- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Phase 5

Adding the units first:

$$\begin{array}{r} 47 \\ +\ \underline{\ 76} \\ 13 \\ \underline{110} \\ 123 \end{array}$$

Discuss how adding the units first give the same answer as adding the tens first. Refine over time to adding the unit digits first consistently.

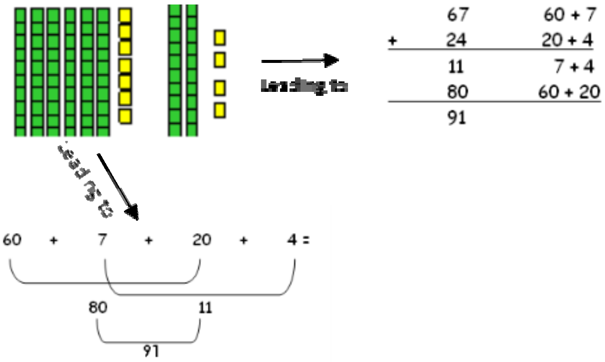
Develop on to adding three digit numbers in vertical layout.

Extend to use decimals to 2 decimal places when ready.

$$\begin{array}{r} 24.68 \\ +\underline{17.94} \\ 0.12 \\ 1.50 \\ 11.00 \\ \underline{30.00} \\ 42.62 \end{array}$$

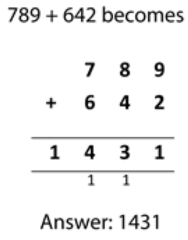
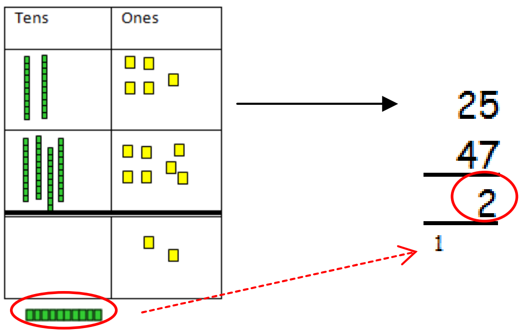
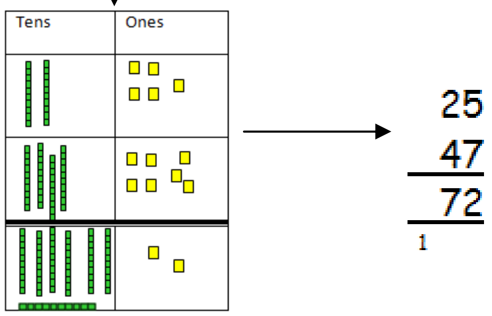
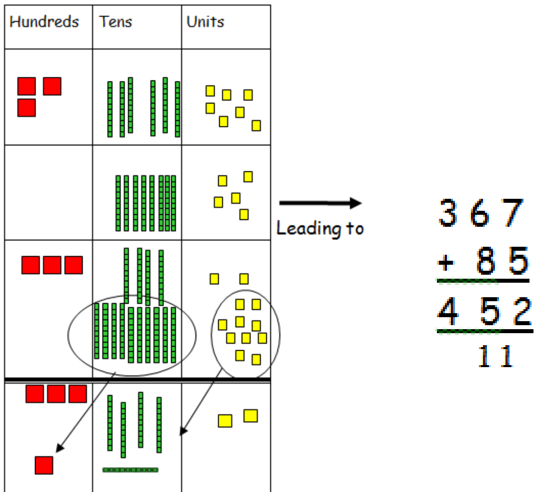
Base 10 equipment:

67 + 24 = 91



As above

Hundreds boundary, approximate

Phase 6: 'Standard' column method/ compact method - In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'. - Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits, including decimals.	**Phase 6** $\begin{array}{r}47\\+\ 76\\\hline 123\\11\end{array}\qquad\begin{array}{r}258\\+\ 87\\\hline 345\\11\end{array}\qquad\begin{array}{r}366\\+458\\\hline 824\\11\end{array}$ Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable. The example from the National Curriculum 2014 given for this phase is:  789 + 642 becomes $\begin{array}{rrrr} & 7 & 8 & 9\\+ & 6 & 4 & 2\\\hline 1 & 4 & 3 & 1\\ & 1 & 1 & \\\hline\end{array}$ Answer: 1431 In this phase the standard method is expanded to include the use of decimals. $\begin{array}{r}45.24\\+\ 26.59\\\hline 71.83\\1\ \ 1\end{array}$ **Compact method**  Leading to  	***As above*** Carry, units boundary, tenths boundary, hundredths boundary.

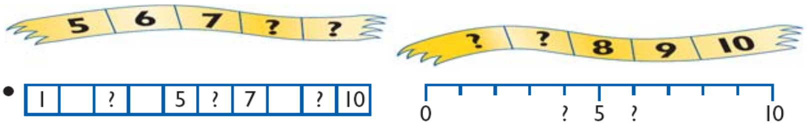
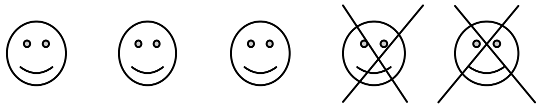
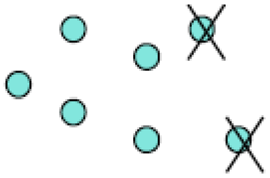
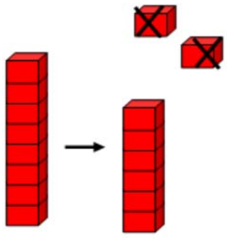
Written methods for subtraction

These phases show the building up to using an efficient / formal method for subtraction of two-digit and three-digit whole numbers.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of hundreds, tens and ones (units) in different ways (e.g. partition 274 into 200 + 70 + 4 or 200 + 60 + 14).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Phase 1 Develop secure one-one correspondence and understanding of subtraction. Taking away (separation model)	• Count accurately forwards and backwards from 0-10 • Recognise and write numerals 1-10  What is the number before 5? And after 5? Before 10? What is the number between 3 and 5? • What numbers are between 7 and 10? • Count and subtract sets of real objects and pictures. 5-2 = 3  Where one quantity is taken away from another to calculate what is left. 7 – 2 = 5  Multilink towers - to physically take away objects. 

Phase 2 – number line and 100 square

The number line helps children to move on from using concrete objects, in a similar way to addition.

Children begin to split a number to subtract back to the nearest multiple of 10 and then count on.

The 100 square supports children's understanding when subtracting ten from any number the units stay the same and the tens digit increases. Eventually this will be done mentally.

Children may need to use concrete resources to secure this. This could be done using a bead strings or number tracks.

Phase 2

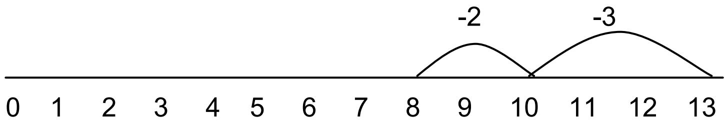
Number line

To be able to subtract/take away one less on a number line.

8-1 = 7

To be able to subtract through 10

13-5 = 8



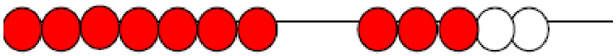
To be able to subtract 10 from any number up to 100 using a 100 square.

34-10=24 (Children notice that 10 less is the number directly above).

To be able to subtract multiples of 10 from any number up to 100 using a 100 square.

94 – 40 = 54

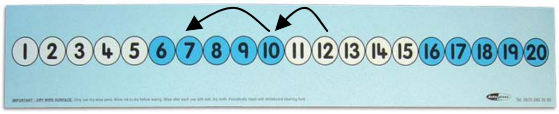
Bead string:



12 – 7 is decomposed / partitioned in 12 – 2 – 5.

The bead string illustrates 'from 12 how many to the last/previous multiple of 10?' and then 'if we have used 2 of the 7 we need to subtract, how many more do we need to count back? (ability to decompose/partition all numbers applied)

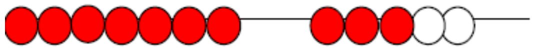
Number Track:



Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Counting up or 'Shop keepers' method

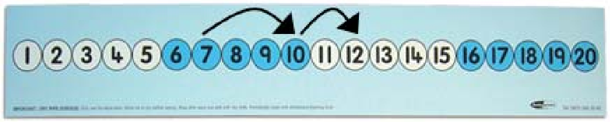
Bead string:

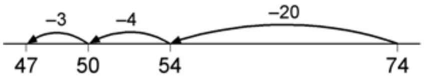
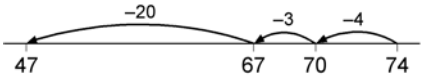
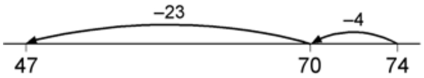
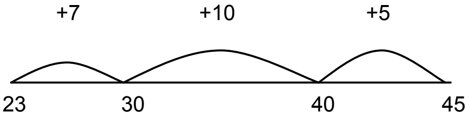


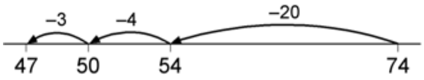
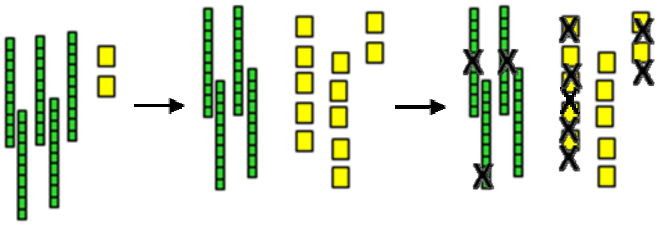
12 – 7 becomes 7 + 3 + 2.

Starting from 7 on the bead string 'how many more to the next multiple of 10?' (children should recognise how their number bonds are being applied), 'how many more to get to 12?'.

Number Track:



Phase 3: Using the empty number line • The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten. • The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47. • With practice, children will need to record less information and decide whether to count back or forward. • Children need to understand that counting back from the larger number and counting up to find the difference will both give the same answer.	**Phase 3** Steps in subtraction can be recorded on an empty number line. The steps often bridge through a multiple of 10. 15 – 7 = 8  74 – 27 = 47 worked by counting back:  The steps may be recorded in a different order:  or combined:  Subtract a 2 two digit number by finding the difference through counting on an empty number line. 45 - 23=  7+ 10 + 5 = 22 45 – 23 = 22 It is important to ask children whether counting up or back is the more efficient for calculations. E.g. 57 – 12 (Is it quicker to count back 12 from 52?), 86 – 77 (Is it quicker to count up from 77 to 86 to find the difference or to count back 77?).	Number line, subtract, count back, count up / find the difference

Phase 4: Partitioning • Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For 74 – 27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 4 in turn. Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.	**Phase 4** Subtraction can be recorded using partitioning: 74 – 27 = 74 – 20 – 7 = 54 – 7 = 47 This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line.  <u>Base 10 equipment:</u> 52 – 37 = 15 	Subtraction, difference, minus, more than,

Phase 5: Expanded layout, leading to

Phase 6: Formal column method

- Partitioning the numbers into hundreds, tens and units and writing one number under the other mirrors the column method, where units are placed under units and tens under tens.
- This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.
- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

Phase 5: (Leading to Phase 6 on the right)

Partitioned numbers are then written under one another:

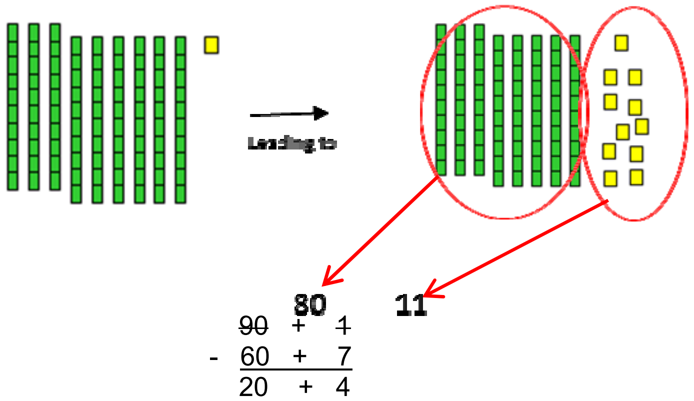
Example: 74 − 27

$$\begin{array}{r} 70+4 \\ -\ 20+7 \\ \hline \end{array} \qquad \begin{array}{r} \overset{60}{\cancel{70}}+\overset{14}{\cancel{4}} \\ -\ 20+7 \\ \hline 40+7 \end{array} \qquad \begin{array}{r} \overset{6}{\cancel{7}}\ \overset{14}{\cancel{4}} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array}$$

Example: 741 − 367

$$\begin{array}{r} 700+40+1 \\ -\ 300+60+7 \\ \hline \end{array} \qquad \begin{array}{r} \overset{600}{\cancel{700}}+\overset{130}{\cancel{40}}+\overset{11}{\cancel{1}} \\ -\ 300+60+7 \\ \hline 300+70+4 \end{array} \qquad \begin{array}{r} \overset{6}{\cancel{7}}\ \overset{13}{\cancel{4}}\ \overset{11}{\cancel{1}} \\ -\ 3\ 6\ 7 \\ \hline 3\ 7\ 4 \end{array}$$

91 – 67 = 24



$$\begin{array}{r} \overset{80}{\cancel{90}}+\overset{11}{\cancel{1}} \\ -\ 60+7 \\ \hline 20+4 \end{array}$$

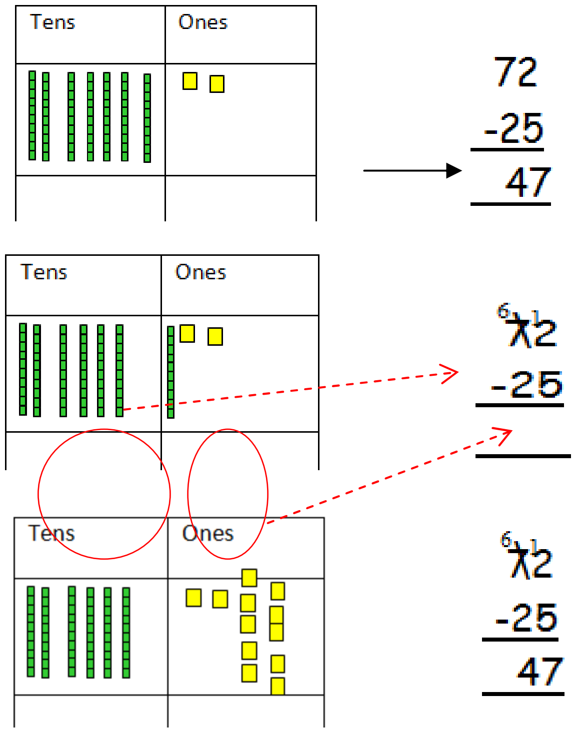
Phase 6:

An example for a formal written subtraction in the National Curriculum 2014:

932 – 457 becomes

$$\begin{array}{r} \overset{8}{\cancel{9}}\ \ \overset{12}{\cancel{3}}\ \ \overset{1}{2} \\ -\ 4\ \ 5\ \ 7 \\ \hline 4\ \ 7\ \ 5 \\ \hline \end{array}$$

Answer: 475



$$\begin{array}{r} 72 \\ -25 \\ \hline 47 \\ \hline \end{array} \qquad \begin{array}{r} \overset{6}{\cancel{7}}{}^{1}2 \\ -25 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} \overset{6}{\cancel{7}}{}^{1}2 \\ -25 \\ \hline 47 \\ \hline \end{array}$$

exchange

Vertical acceleration

By returning to earlier manipulative experiences children are supported to make links across mathematics, encouraging 'If I know this…then I also know…' thinking.

Decimals

Ensure that children are confident in counting forwards and backwards in decimals – using bead strings to support.

Bead strings:

Each bead represents 0.1, each different block of colour equal to 1.0

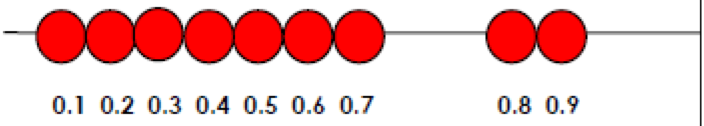
Base 10 equipment:

Addition of decimals

Aggregation model of addition

Counting both sets – starting at zero.

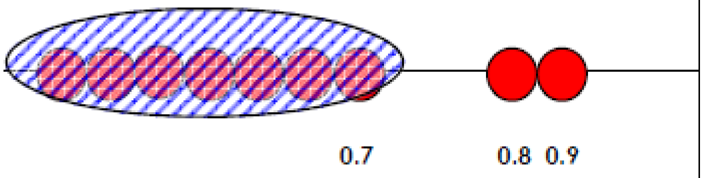
0.7 + 0.2 = 0.9



Augmentation model of addition

Starting from the first set total, count on to the end of the second set.

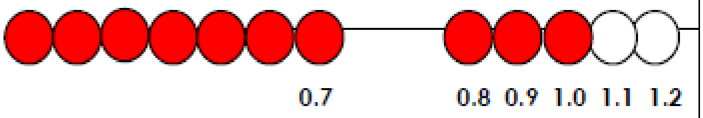
0.7 + 0.2 = 0.9



Bridging through 1.0

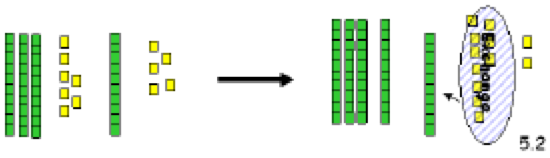
Encouraging connections with number bonds.

0.7 + 0.5 = 1.2



Partitioning

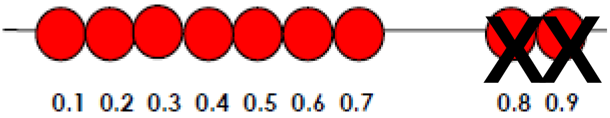
3.7 + 1.5 = 5.2



Subtraction of decimals

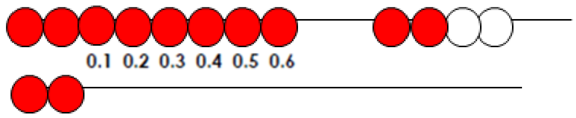
Take away model

0.9 – 0.2 = 0.7



Finding the difference (or comparison model):

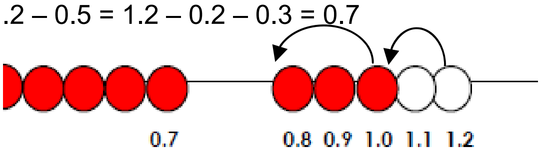
0.8 – 0.2 =



Bridging through 1.0

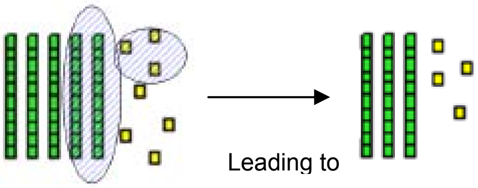
Encourage efficient partitioning.

1.2 – 0.5 = 1.2 – 0.2 – 0.3 = 0.7



Partitioning

5.7 – 2.3 = 3.4



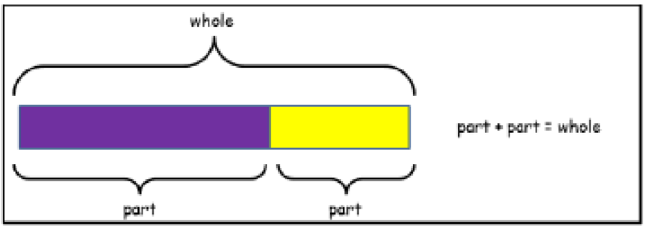
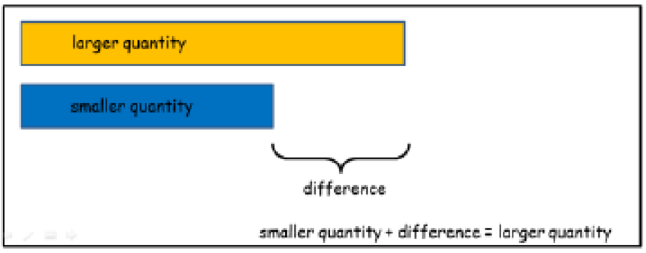
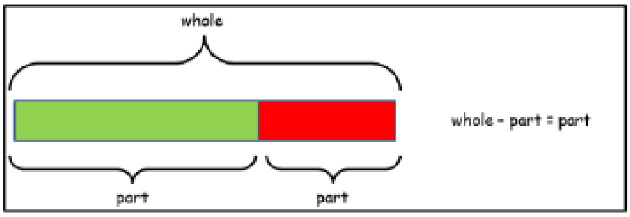
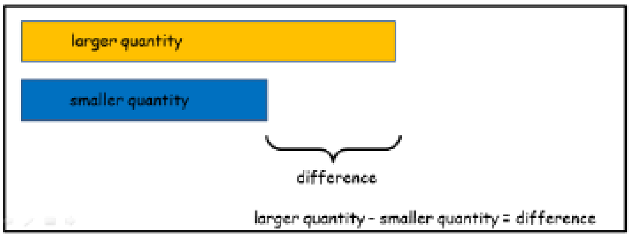
Gradation of difficulty- addition:

Gradation of difficulty- subtraction:

1. No exchange 2. Extra digit in the answer 3. Exchanging ones to tens 4. Exchanging tens to hundreds 5. Exchanging ones to tens and tens to hundreds 6. More than two numbers in calculation 7. As 6 but with different number of digits 8. Decimals up to 2 decimal places (same number of decimal places) 9. Add two or more decimals with a range of decimal places	1. No exchange 2. Fewer digits in the answer 3. Exchanging tens for ones 4. Exchanging hundreds for tens 5. Exchanging hundreds to tens and tens to ones 6. As 5 but with different number of digits 7. Decimals up to 2 decimal places (same number of decimal places) 8. Subtract two or more decimals with a range of decimal places

Solving problems involving addition and subtraction

This method can be used as an additional support to help children solve problems in familiar practical contexts, including using quantities. Problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

Singapore Bar Method: This method can be used to support children's understanding of: • Solving problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = – 9. • Solving problems, including missing number problems, using number facts, place value, and more complex addition and subtraction. • Solving addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.	   	put together, add, altogether, total, take away, distance between, difference between, more than and less than

Written methods for multiplication of whole numbers

These phases show the stages in building up to using an efficient method for multiplication

To multiply successfully, children need to be able to:

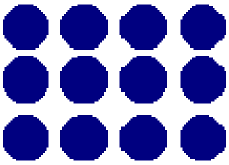
- recall all multiplication facts to 10 × 10; (The expectation in the National Curriculum 2014 is that children can recall up to 12 x 12 by the end of Year 4).
- partition number into multiples of hundreds, tens and ones (units);
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

Progression in Multiplication and Division

Multiplication and division are connected.
Both express the relationship between a number of equal parts and the whole.

Part	Part	Part	Part
Whole			



The following array, consisting of four columns and three rows, could be used to represent the number sentences: -

3 x 4 = 12,

4 x 3 =12,

3 + 3 + 3 + 3 = 12,

4 + 4 + 4 =12.

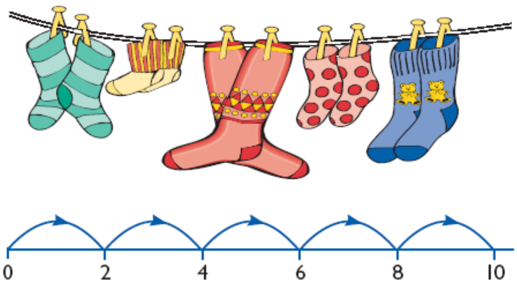
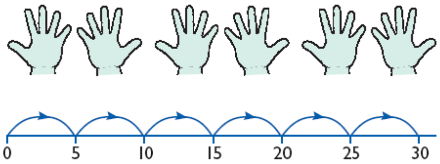
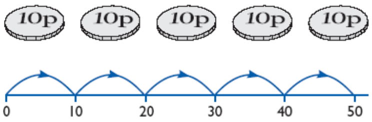
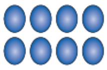
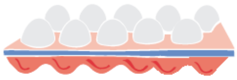
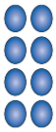
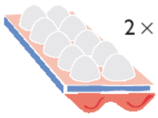
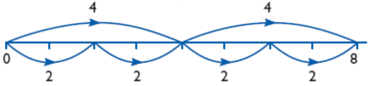
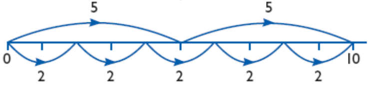
And it is also a model for division

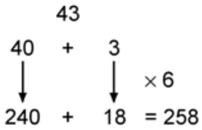
12 ÷ 4 = 3

12 ÷ 3 = 4

12 – 4 – 4 – 4 = 0

12 – 3 – 3 – 3 – 3 = 0

Phase 1: Hands on experiences Initially children put objects into groups/ sets. Solve multiplication through repeated addition.	 $2 + 2 + 2 + 2 + 2 = 10$ $2 \times 5 = 10$ 2 multiplied by 5 5 pairs 5 hops of 2  $5 + 5 + 5 + 5 + 5 + 5 = 30$ $5 \times 6 = 30$ 5 multiplied by 6 6 groups of 5 6 hops of 5  $10p + 10p + 10p + 10p + 10p = 50p$ $10p \times 5 = 50p$ 5 hops of 10
To use arrays to solve multiplication problems.	 $4 \times 2 = 8$ $2 \times 4 = 8$  $5 \times 2 = 10$ $2 \times 5 = 10$  $2 \times 4 = 8$ $4 \times 2 = 8$  $2 \times 5 = 10$ $5 \times 2 = 10$
Children begin to identify patterns within multiplications (e.g. number always ends with a 0, 2, 4, 6 or 8 in the 2X table) and between multiplications (4X table is double the 2X table).	2 hops of 4  4 hops of 2 2 hops of 5  5 hops of 2 To multiply by 4 17x4= 17x2x2 = 34x2 = 68 To multiply by 5 14x5= (14x10) ÷ 2= 140 ÷ 2 =70

Phase 2: Mental multiplication using partitioning • Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and units to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the units can be multiplied first but it is more common to start with the tens.	**Phase 2** Informal recording in might be:  Also record mental multiplication using partitioning: $14 \times 3 = (10 + 4) \times 3$ $= (10 \times 3) + (4 \times 3) = 30 + 12 = 42$ $43 \times 6 = (40 + 3) \times 6$ $= (40 \times 6) + (3 \times 6) = 240 + 18 = 258$ Note: These methods are based on the distributive law. Children should be introduced to the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7: ○○○○○○○ ○○○○○…○○ ○○○○○○○ ○○○○○…○○ ○○○○○○○ ○○○○○…○○ $7 \times 3 = (5 + 2) \times 3 = (5 \times 3) + (2 \times 3) = 15 + 6 = 21$	**Key Vocabulary** product multiplication, array, partition,

Phase 3: The grid method

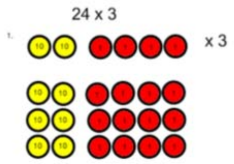
- As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.
- When working with larger numbers, it is usually better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

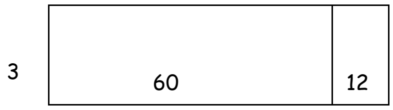
  Grid method can be used for multiplying numbers with various amounts of digits, including decimal numbers e.g. 15 x 3.5 or 142 x 27.

Phase 3

To know how the grid method is based on arrays and extend the use of arrays to the grid method.

3x24 = (3x20) + (3x4) = 72





Using grid method:

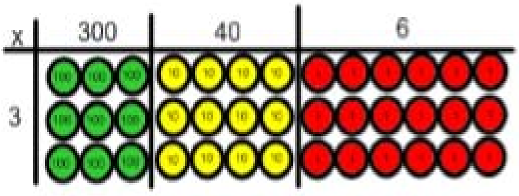
38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266

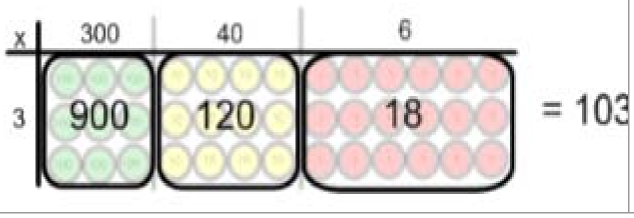
×	7
30	210
8	56
	266

Many children find grid method supports their mental multiplication, as it helps to structure thinking.

286 x 29 = 8294

×	20	9	
200	4000	1800	5800
80	1600	720	2320
6	120	54	174
			8294
			1

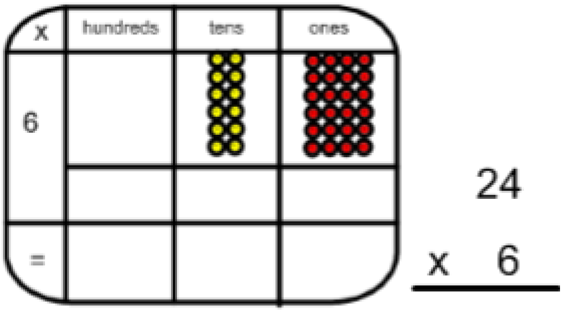
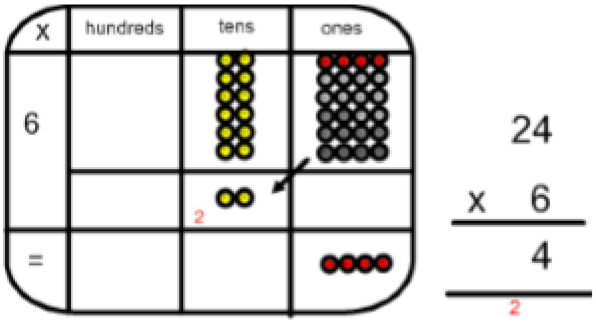




As above

grid method,
partition,
multiplication,
column.

Phase 4: Expanded / long multiplication	**Phase 4**	
• The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above. • Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed.	38 x 7 expanded: $\begin{array}{rlr} 30+8 & & 38 \\ \times\ \ 7 & & \times\ \ 7 \\ \hline 210 & 30\times7=210 & 210 \\ 56 & 8\times7=\ \ 56 & 56 \\ \hline 266 & & 266 \\ \hline \end{array}$ 56 × 27 expanded: (is approximately 60 × 30 = 1800) $\begin{array}{rl} 56 & \\ \times\ \ 27 & \\ \hline 1000 & 50\times20=1000 \\ 120 & 6\times20=\ \ 120 \\ 350 & 50\times7=\ \ 350 \\ 42 & 6\times7=\ \ \ \ 42 \\ \hline 1512 & \\ \hline 1 & \end{array}$ Moving to reduced recording when ready: 56 × 27 is approximately 60 × 30 = 1800. $\begin{array}{rl} 56 & \\ \times\ \ 27 & \\ \hline 1120 & 56\times20 \\ 392 & 56\times\ \ 7 \\ \hline 1512 & \\ \hline 1 & \end{array}$	

Phase 5:Short multiplication • The recording is reduced further, with carry digits recorded below the line. • If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of Phase 3 (Grid method) or Phase 4.	**Phase 5** The array using place value counters becomes the basis for understanding short multiplication first without exchange before moving onto exchanging 24 x 6   $\begin{array}{cr} & 38 \\ \times & \underline{7} \\ & \underline{266} \\ & {\scriptstyle 5} \end{array}$ The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage. **E**xamples for a formal written multiplication in the National Curriculum 2014: Short multiplication: (Phase 5) 342 × 7 becomes $\begin{array}{cccc} & 3 & 4 & 2 \\ \times & & & 7 \\ \hline 2 & 3 & 9 & 4 \\ \hline & {\scriptstyle 2} & {\scriptstyle 1} & \end{array}$ Answer: 2394 Long multiplication: (Phase 4/5) 124 × 26 becomes $\begin{array}{cccc} & {\scriptstyle 1} & {\scriptstyle 2} & \\ & 1 & 2 & 4 \\ \times & & 2 & 6 \\ \hline 2 & 4 & 8 & 0 \\ & 7 & 4 & 4 \\ \hline 3 & 2 & 2 & 4 \\ \hline {\scriptstyle 1} & {\scriptstyle 1} & & \end{array}$ Answer: 3224	**As above** Carry,

Written methods for division

From phase 2 it shows the building up to long division through Years 4 to 6 – first long division TU ÷ U, extending to HTU ÷ U, then HTU ÷ TU, and then short division HTU ÷ U.

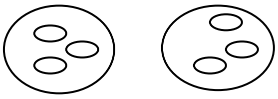
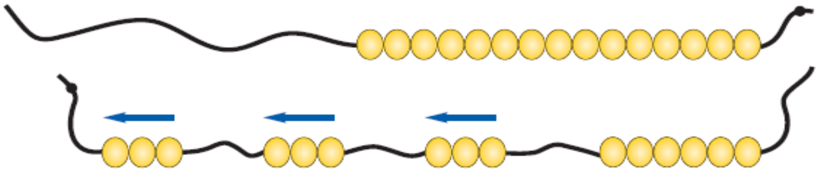
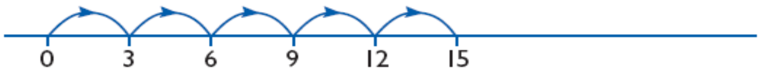
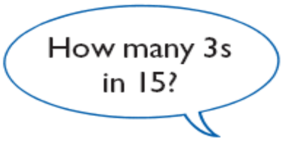
To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out written methods of division successful, children also need to be able to:

- understand division as repeated subtraction;
- estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- multiply a two-digit number by a single-digit number mentally;
- subtract numbers using the column method.

Phase 1	To half numbers to 10/20 – practically and mentally. To be able to divide practically by sharing. 6 eggs shared between 2 nests = 3  
To divide on a number line initially with no remainders and later with remainders.	   $15 \div 3 = 5$  5 hops in 15. How big is each hop? $15 \div 5 = 3$ 15 shared between 5
Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s.	

Phase 2: Mental division using partitioning

- Mental methods for dividing TU ÷ U can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient.
- Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing the correspondence to mental methods of multiplication, have not in the past been given enough attention.
- Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.

Phase 2

One way to work out TU ÷ U mentally is to partition TU into a multiple of the divisor plus the remaining units, then divide each part separately.

Informal recording in Year 4 for 84 ÷ 7 might be:

84

70 + 14

↓ ↓ ÷ 7

10 + 2 = 12

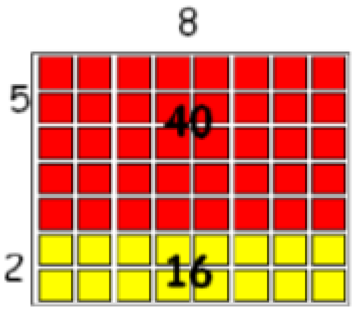
In this example, using knowledge of multiples, the 84 is partitioned into 70 (the highest multiple of 7 that is also a multiple of 10 and less than 84) plus 14 and then each part is divided separately using the distributive law.

For this method to be efficient for the child, they need to have a secure knowledge of the multiplication facts.

Arrays:

The array is also a flexible model for division of larger numbers

56 ÷ 8 = 7



Children could break this down into more manageable arrays, as well as using their understanding of the inverse relationship between division and multiplication.

56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8) = 5 + 2 = 7

Key Vocabulary

Partition, share, divide, multiple

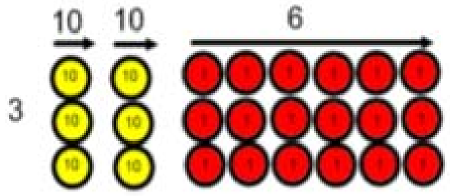
Phase 3: Chunking on a number line

- This method is based on counting up in multiples of the divisor from the number to be divided, the dividend. (10 x 7 = 70, so counting up 10 groups you ‘land’ on 70).
- There is a link to the mental methods for ÷.
- Once they understand and can apply the method, children should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same.

Phase 3

Children continue to use arrays and partitioning where appropriate, to prepare them for the ‘chunking’ and short method of division. Arrays are represented as ‘grids’ as a shorthand version.

e.g. 78 ÷ 3 =

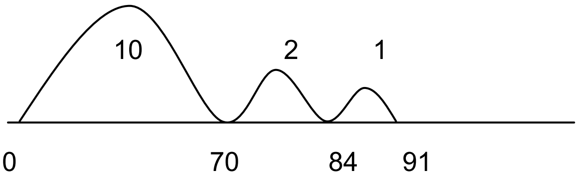


78 ÷ 3 = (30 ÷ 3) + (30 ÷ 3) + (18 ÷ 3) =

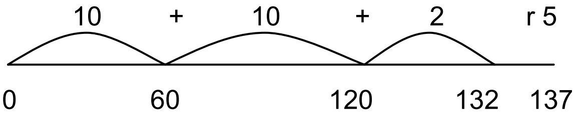
10 + 10 + 6 = 26

To divide on a number line by chunking.

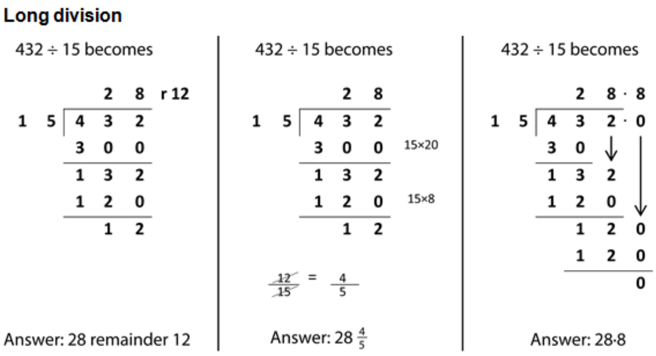
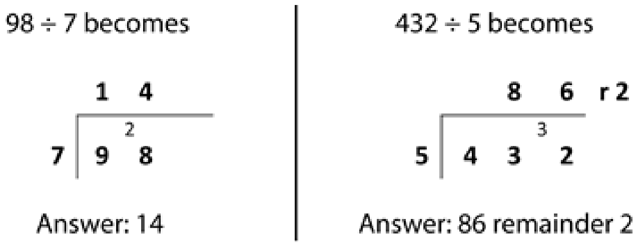
91 ÷ 7 =13 (10x7, 2x7, 1x7)



137 ÷ 6 = 22 r 5 (10x6, 10x6, 2x6, r5)



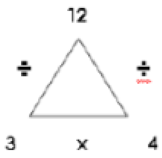
Phase 4: 'Chunking' method • This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract. • Chunking is useful for reminding children of the link between division and repeated subtraction. It also becomes a mental method for many pupils, linking closely to long division. • However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. – to help to choose a starting point for the division; – to check the answer after the calculation.	$\begin{array}{rr} 6\overline{)196} & \\ -\underline{\ 60} & 6\times10 \\ 136 & \\ -\underline{\ 60} & 6\times10 \\ 76 & \\ -\underline{\ 60} & 6\times10 \\ 16 & \\ -\underline{\ 12} & 6\times\underline{\ 2} \\ 4 & 32 \\ \text{Answer:} & 32\,\text{R}\,4 \end{array}$ It is important to encourage children to estimate first e.g. 196 ÷ 6 = 60 is 10 x6, so 120 is 20 x 6, So, 180 is 30 x 6 and 240 is 40 x 6 The answer will be between 30 and 40 Encourage children to reduce the number of steps and move them on quickly to finding the largest possible multiples. $\begin{array}{rr} 6\overline{)196} & \\ -\underline{180} & 6\times30 \\ 16 & \\ -\underline{\ 12} & 6\times\underline{\ 2} \\ 4 & 32 \\ \text{Answer:} & 32\,\text{R}\,4 \end{array}$ Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.	
Phase 5: Long division The next step is to tackle HTU ÷ TU. The layout on the right, which links to chunking, is in essence the 'long division' method. Recording the build-up to the quotient on the left of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient. Conventionally the 20, or 2 tens, and the 3 units forming the answer are recorded above the line, as in the second recording.	**Phase 5** How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560. $\begin{array}{rr} 24\overline{)\ 560} & \\ -\underline{480} & 24\times20 \\ 80 & \\ \underline{72} & 24\times3 \\ 8 & \end{array}$ Answer: 23 R 8 In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below. $\begin{array}{r} 23 \\ 24\overline{)\ 560} \\ -\underline{480} \\ 80 \\ -\underline{72} \\ 8 \end{array}$ Answer: 23 R 8	

Phase 5 (cont): Long division Long division can be used to work out HTU ÷ TU. The method is similar to 'Chunking' method. The answer give a remainder, decimal or fraction.	Examples for long division in the National Curriculum 2014:  These show how long multiplication builds on 'Chunking' method and can be used to find remainders or decimal fractions.	
Phase 6: Short division	Examples for short division in the National Curriculum 2014:  Short division can also be used to find a decimal answer. 0 4 6 . 4 7) $3\ {}^{3}2\ {}^{4}4.\ {}^{2}8$	

Inverse operations

Trios can be used to model the 4 related multiplication and division facts. Children learn to state the 4 related facts.

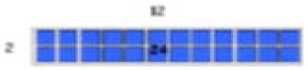
3 x 4 = 12
4 x 3 = 12
12 ÷ 3 = 4
12 ÷ 4 = 3

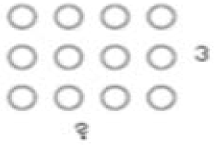


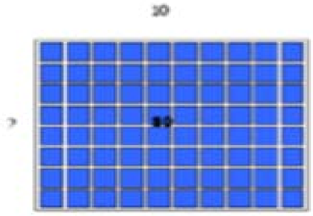
Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

□ x 5 = 20   3 x ∆ = 18   O x □ = 32
24 ÷ 2 = □   15 ÷ O = 3   ∆ ÷ 10 = 8

This can also be supported using arrays: e.g. 3 X ? = 12

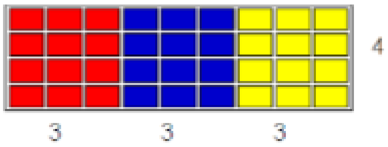






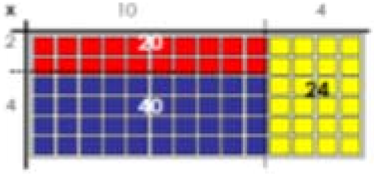
To be successful in calculation learners must have plenty of experiences of being flexible with partitioning, as this is the basis of distributive and associative law.

Associative law (multiplication only) :-
E.g. 3 x (3 x 4) = 36

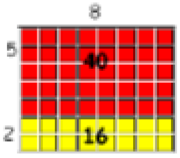


The principle that if there are three numbers to multiply these can be multiplied in any order.

Distributive law (multiplication):-
E.g. 6 x 14 = (2 x 10) + (4 x 10) + (4 x 6) = 20 + 40 + 24 = 84
This law allows you to distribute a multiplication across an addition or subtraction.

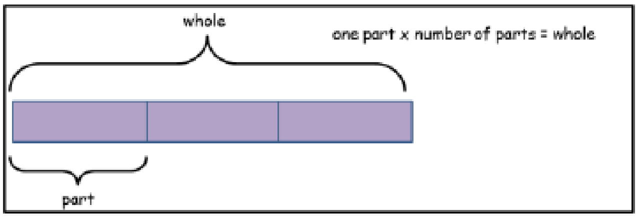
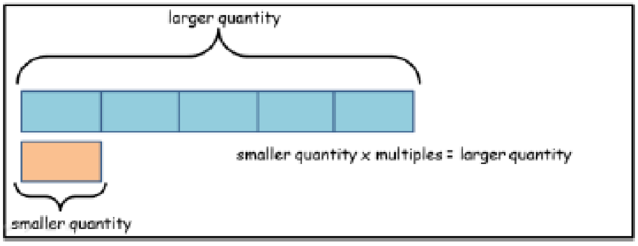
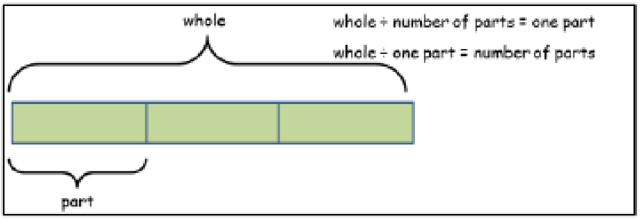
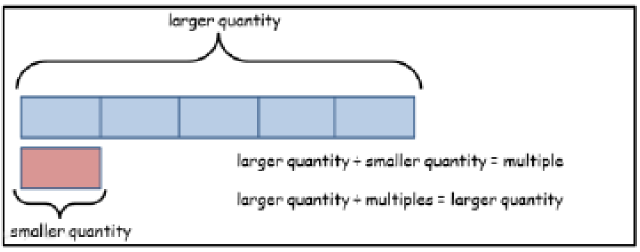


Distributive law (division):-
E.g. 56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8) = 5 + 2 = 7
This law allows you to distribute a division across an addition or subtraction.



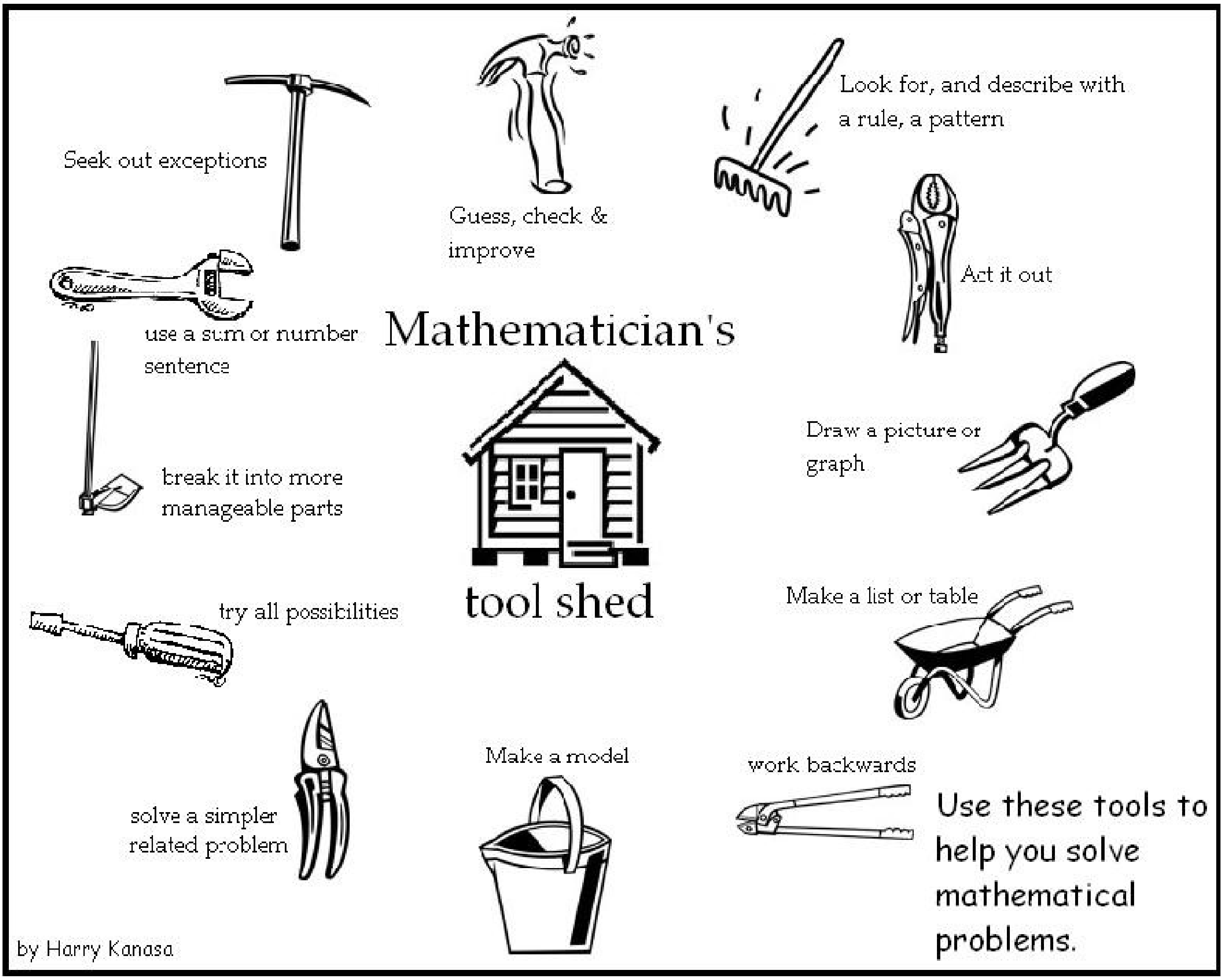
Solving problems involving multiplication and division

This method can be used as an additional support to help children solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Singapore Bar Method: This method can be used to support children's understanding of: - Solving one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher. - Solving problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects. - Solving problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.	   	Multiplication, division, doubling, finding simple fractions of objects, partition, share, divide, multiple

For a compact version of this Calculations policy, please also see the 'Calculations Policy Quick Reference Guide', which summarises the methods taught across the school.

Any questions about the above content can be discussed with the Maths Subject Leader or members of the Senior Leadership Team.



Mathematician's tool shed

Seek out exceptions

Guess, check & improve

Look for, and describe with a rule, a pattern

Act it out

use a sum or number sentence

break it into more manageable parts

Draw a picture or graph

try all possibilities

Make a list or table

solve a simpler related problem

Make a model

work backwards

Use these tools to help you solve mathematical problems.

by Harry Kanasa

Which learning habit did you use?

Communication	☐	**Curiosity**	☐
Independence	☐	**Determination**	☐

I know …. because …..

Because I know …., I now know ……

If …. then ….

This is different because ….

This is always true because …..

This is sometimes true because ….

This is the same because ….

I used … to help me.